TOPIC - Problem based on Linear Differential Equation

B.Sc. (II)

Date -

By - Samrend...

Equation of the form

$$\frac{dx}{dy} + Px = Q.$$

Where P, Q are the function of y

STEP I To Calculate

I.F $= e^{\int P\,dy}$

STEP II - $e^{\int P\,dy} \cdot \frac{dx}{dy} + e^{\int P\,dy} \cdot x = Q e^{\int P\,dy}$

STEP III $\frac{d\, x e^{\int P\,dy}}{dy} = Q e^{\int P\,dy}$

$\Rightarrow x e^{\int P\,dy} = \int Q e^{\int P\,dy}\, dy$

Gives the solution of the required Equation

Problem 1

Solve $(x+y+1)\frac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{x+y+1}$$

$$\Rightarrow \frac{dx}{dy} = x+y+1$$

$$\Rightarrow \frac{dx}{dy} - x = y+1$$

which is of the form

$$\frac{dx}{dy} + Px = Q.$$

$$I.F = e^{-\int dy} = e^{-\cancel{y}} = e^{-y}.$$

$$\therefore \frac{d(xe^{-y})}{dy} = e^{-y}(y+1)$$

$$\Rightarrow \int d(xe^{-y}) = \int e^{-y}(y+1)\,dy$$

$$\Rightarrow xe^{-y} = \int y e^{-y}\,dy + \int e^{-y}\,dy$$

$$\Rightarrow xe^{-y} = y\int e^{-y}\,dy + \int\left\{\int e^{-y}dy \cdot \frac{dy}{dy}\right\}dy \quad \frac{e^{-y}}{-1}$$

$$= -ye^{-y} \cancel{+} \int e^{-y}\,dy - e^{-y}$$

$$= -ye^{-y} \cancel{+}\, \cancel{e^{-y}} - \cancel{e^{-y}} + c$$

$$\Rightarrow xe^{-y} = -2y e^{-y} + c$$

$$\Rightarrow x = -2y + \frac{c}{e^{-y}}$$

$$\Rightarrow x + 2y = ce^{y}$$ which is the required solution

Ex:-> $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

$$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$I.F = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

$$\Rightarrow \frac{d\, x e^{\tan^{-1}y}}{dy} = \frac{e^{\tan^{-1}y} \cdot \tan^{-1}y}{1+y^2}$$

$$\int d(x e^{\tan^{-1}y}) = \int \frac{e^{\tan^{-1}y} \cdot \tan^{-1}y}{1+y^2}\,dy$$

$$\Rightarrow x e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y} \cdot \tan^{-1}y}{1+y^2}\,dy$$

put $\tan^{-1}y = z$

$$\frac{1}{1+y^2}dy = dz$$

$$= \int z e^z\, dz$$

$$= z e^z - e^z$$

$$x e^{\tan^{-1}y} = \tan^{-1}y\, e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$$

which is the required so